Level – 1	**Daily Tutorial Sheet - 3**

31.(A) $I=\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx=\int \frac{\sqrt{\tan x}}{\tan x}\sec^2 x\,dx=\int \frac{1}{\sqrt{t}}dt$, where $t=\tan x$

$I=2t^{1/2}+C=2\sqrt{\tan x}+C$

32.(C) Putting $5^{5^{5^x}}=t$, we have $5^{5^{5^x}}5^{5^x}5^x(\log 5)^3\,dx=dt$,

we get: $\int 5^{5^{5^x}}.5^{5^x}.5^x\,dx=\frac{1}{(\log 5)^3}\int 1\,dt=\frac{t}{(\log 5)^3}+C=\frac{5^{5^{5^x}}}{(\log 5)^3}+C$

33.(C) Putting $\sin x=t$ in the given integral, we get: $\int \frac{1-t^2+\left(1-t^2\right)^2}{t^2+t^4}\,dt=\int \frac{\left(1-t^2\right)\left(2-t^2\right)}{t^2+t^4}\,dt$

$=\int\left(1+\frac{2}{t^2}-\frac{6}{1+t^2}\right)dt=t-\frac{2}{t}-6\tan^{-1}(t)+C=\sin x-2(\sin x)^{-1}-6\tan^{-1}(\sin x)+C$

34.(C) The anti-derivative of $f(x)=e^{x/2}$ is given by $g(x)=\int f(x)=\int e^{x/2}dx=2e^{x/2}+C$ .... (i)

The curve $y=g(x)$ passes through the point (0, 3). $\therefore$ $3=3e^0+C \Rightarrow C=1$

Putting C = 1 in (i), we get: $g(x)=2e^{x/2}+1$

35.(A) We have $f(x)\int \tan^{-1}\sqrt{x}\,dx+C=x\tan^{-1}\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}}{1+x}dx+C=x\tan^{-1}\sqrt{x}-\frac{1}{2}\int\frac{2t^2}{1+t^2}dt+C$, where $x=t^2$

$=x\tan^{-1}\sqrt{x}-\int\frac{t^2+1-1}{t^2+1}dt+C=x\tan^{-1}\sqrt{x}-t+\tan^{-1}t+C=x\tan^{-1}\sqrt{x}-\sqrt{x}+\tan^{-1}\sqrt{x}+C$

Since $y=f(x)$ passes through (0, 2). Therefore, $C=2$.

Hence, $f(x)=x\tan^{-1}\sqrt{x}-\sqrt{x}+\tan^{-1}\sqrt{x}+2$

36.(C) $f(x)=\int x\sqrt{1-x^2}\,dx=\int\frac{-1}{2}\times(-2x)\sqrt{1-x^2}\,dx$

Put $1-x^2=t$ to get: $f(x)=\frac{-1}{2}\times\frac{2}{3}t^{3/2}+C=\frac{-1}{3}\left(1-x^2\right)^{3/2}+C$; $f(0)=\frac{7}{3} \Rightarrow C=\frac{8}{3}$

37.(C) Let e^x+C_1 and e^x+C_2 be the two antiderivatives.

Then difference between them is C_1-C_2 which is fixed and equal to 2.

38.(B) Differentiate on both sides to get $f(x)=\frac{\sin\sqrt{x}}{\sqrt{x}}$

39.(D) $I=\int\frac{\cos 4x-1}{\cot x-\tan x}dx=\int\left(\frac{-2\sin^2(2x)}{\cos^2 x-\sin^2 x}\right)\cdot\sin x\cdot\cos x\,dx$

$=-\int\frac{1-\cos^2(2x)}{\cos(2x)}\cdot\sin(2x)dx=\frac{1}{2}\int\frac{1-t^2}{t}.\,dt=\frac{1}{2}\left(\log|t|-\frac{t^2}{2}\right)+C=\frac{1}{2}\log|\cos 2x|-\frac{\cos^2(2x)}{4}+C$

40.(AB) Put $\sin^2 x=t \Rightarrow 2\sin x\cdot\cos x\,dx=dt$ $\Rightarrow I=\int\frac{dt}{\sqrt{1-t^2}}=\frac{1}{2}\sin^{-1}(t)+C=\frac{1}{2}\sin^{-1}\left(\sin^2 x\right)+C$

41.(B) $\displaystyle\int \frac{\sin x \cos x \, dx}{1+(\sin x+\cos x)} \cdot \frac{1-(\sin x+\cos x)}{1-(\sin x+\cos x)}$

$$= \int \frac{(1-\sin x-\cos x)\sin x \cos x \, dx}{1-1-2\sin x\cos x} = -\frac{1}{2}\int (1-\sin x-\cos x)\,dx = \frac{1}{2}(\sin x-\cos x-x)+C$$

42. **(i)** Let $x = A(2x+1)+B$ and proceed **(ii)** Let $3x-2 = A(2x+4)+B$ and proceed

(iii) Let $x+1 = A(2x+1)+B$ and proceed **(iv)** Divide N by D and proceed

(v) Divide N and D by $\cos^2 x$ and proceed

43. **(i)** Put $\cos x = \dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}$

(ii) Let $3\sin x + 2\cos x = A(3\cos x + 2\sin x) + B(-3\sin x + 2\cos x)$ and proceed

(iii) $2 + 3\cos x = \ell(\sin x + 2\cos x + 3) + m(\cos x - 2\sin x) + n$

Comparing coefficients of *sinx*, *cosx* and const. term,we get :

$2 = 3\ell + n$; $3 = 2\ell + m$; $\ell - 2m = 0$ $\Rightarrow$ $m = \frac{3}{5}$; $\ell = \frac{6}{5}$; $n = -\frac{8}{5}$

$$\Rightarrow \int \frac{3\cos x+2}{\sin x+2\cos x+3}\,dx = \frac{6}{5}\int dx + \frac{3}{5}\int \frac{\cos x-2\sin x}{\sin x+2\cos x+3}\,dx - \frac{8}{5}\int \frac{dx}{\sin x+2\cos x+3}$$

$$= \frac{6}{5}x + \frac{3}{5}\log|\sin x+2\cos x+3| - \frac{8}{5}\int \frac{dx}{\sin x+2\cos x+3}$$

Let $I_1 = \displaystyle\int \frac{dx}{\sin x + 2\cos x + 3}$

Put $\sin x = \dfrac{2\tan x/2}{1+\tan^2 x/2}$; $\cos x = \dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}$ and Let $\tan x/2 = t$

$$\Rightarrow I_1 = \int \frac{dx}{\sin x+2\cos x+3} = \int \frac{\sec^2 x/2\,dx}{2\tan\frac{x}{2}+2-2\tan^2\frac{x}{2}+3+3\tan^2 x/2} = \int \frac{\sec^2 x/2\,dx}{\tan^2\frac{x}{2}+2\tan\frac{x}{2}+5}$$

Substitute $\tan\frac{x}{2} = t$ $\Rightarrow$ $\sec^2\frac{x}{2}dx = 2dt$ $\Rightarrow$ $I_1 = \tan^{-1}\left(\dfrac{1+\tan x/2}{2}\right)+c$

(iv) $\displaystyle\int \frac{dx}{(2\sin x+3\cos x)^2} = \int \frac{dx}{(4\sin^2 x+9\cos^2 x+12\sin x\cos x)}$

Divide N and D by $\cos^2 x$ and proceed

(v) $\displaystyle\int \frac{dx}{\cos x(\sin x+2\cos x)} = \int \frac{dx}{\cos x\sin x+2\cos^2 x}$

Divide N and D by $\cos^2 x$ and proceed

44. **(i)** $\displaystyle\int \frac{x^2dx}{x^4+1} = \frac{1}{2}\int \frac{2x^2dx}{1+x^4} = \frac{1}{2}\int \frac{x^2+1}{1+x^4}dx + \frac{1}{2}\int \frac{x^2-1}{1+x^4}dx$

(ii) $\displaystyle\int \frac{dx}{x^4+1} = \frac{1}{2}\int \frac{2dx}{1+x^4} = \frac{1}{2}\int \frac{1+x^2}{1+x^4}dx + \frac{1}{2}\int \frac{1-x^2}{1+x^4}dx$

(iii) $\int \sqrt{\tan x}\, dx = \int \frac{\sqrt{\tan x}\,.\sec^2 x}{1+\tan^2 x} dx = \int \frac{t\,.2t}{1+t^4} dt$ [Put $\tan x = t^2 \Rightarrow \sec^2 x\,.\,dx = 2t dt$]

$= \int \frac{t^2+1}{1+t^4} dt + \int \frac{t^2-1}{t^4+1} dt$

(iv) Substitute $\cot x = t^2$ and solve.

(v) $\int \frac{dx}{\sin^4 x + \cos^4 x} = \int \frac{\sec^4 x dx}{1+\tan^4 x} = \int \frac{(1+\tan^2 x)\sec^2 x}{(1+\tan^4 x)} dx$ [Put $\tan x = t \Rightarrow sec^2 x dx = dt$]

$= \int \frac{(1+t^2).dt}{1+t^4}$ Divide N and D by t^2 and proceed.

45. **(i)** Apply By-parts taking $\log(1 + x)$ as first part.

$$\int x \log(x+1) dx = \frac{x^2}{2}\log(x+1) - \frac{1}{2}\log(x+1) + \frac{x}{2} - \frac{x^2}{4} + c$$

(ii) Apply By-parts taking $\log x$ as first part. $\int \frac{\log x}{x^2} dx = -\frac{\log x}{x} - \frac{1}{x} + c$

(iii) $\int \frac{x}{1+\sin x} dx = \int \frac{x(1-\sin x)}{\cos^2 x} dx = \int x \sec^2 x dx - \int x \tan x \sec x dx$

Apply By - parts taking 'x' as the first part in both the integrations.

$= x\tan x - \ell n\,|\sec x| - (x\sec x - \ell n\,|\sec x + \tan x|) + c$

(iv) Apply By - parts taking $\cos(\log x)$ as first part and dx as second part.

$$\Rightarrow \int \cos(\log x) dx = \frac{x}{2}(\cos(\log x) + \sin(\log x)) + c$$

(v) Apply By - parts taking $\log\left(x + \sqrt{x^2+a^2}\right)$ as first part and dx as second part.

(vi) Put $logx = t \Rightarrow x = e^t \Rightarrow dx = e^t\,.\,dt$

$$\Rightarrow I = \int \frac{\log x}{(1+\log x)^2} dx = \int \frac{t\,.e^t}{(1+t)^2} dt = \int e^t\left[\frac{1}{1+t} - \frac{1}{(1+t)^2}\right] dt = \frac{e^t}{t+1} + c = \frac{x}{1+\log x} + c$$